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Vibration Analysis of Cantilever Smart Structure by using Piezoelectric Smart Material

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Abstract- The field of smart structures and smart materials has been an emerging area of research for last few decades. A smart structure would be able to sense the vibration and generate a controlled actuation to it, so the vibration can be minimized. For this purpose, smart materials are used as actuators and sensors. In this paper, some literature review is given about smart structure and smart material. Piezoelectric material is used as smart material and cantilever beam is considered as a smart structure. Different positions are considered for the model analysis. In this case, the modal analysis are found out by using ANSYS and MATLAB.

Index terms: Smart structure, Smart materials, vibration analysis, vibration control, cantilever beam, cantilever plate.

I. INTRODUCTION

Smart structures is a rapidly advancing field with the range of support and enabling technologies having significant advances, notable optics and electronics. The definition of smart structure was a topic of controversy from the late 1970 to 1980. In order to define this a special workshop was organized by the US army research office in 1988 in which Sensors, Actuators, Control mechanism and Timely response were recognized as the four qualifying features of any smart system or structures.

In this workshop Smart structure is defined as “A system or material which has built in intrinsic Sensor, actuator and control mechanism whereby it is capable of sensing a stimulus, responding to it in a predetermined manner and extent, in a short time and reverting to its original state as soon as the stimulus is removed.”

According to Spilman a smart structure is defined as “a physical structure having a definite purpose, means of imperactive to achieve that purpose and the pattern of functioning of a computer.”

Smart structure contains a host structure, a sensor to gauge its internal state, an actuator to affect its internal state and, a controller whose purpose is to process the sensors and appropriately send signals to actuators.

Vibration control is an important area of interest in several industrial applications. Unwanted vibration can have a detrimental and sometimes catastrophic effect on the serviceability or structural integrity of mechanical systems. To control the vibrations in a system, different techniques have been developed. Some of these techniques and methods use piezoelectric materials as sensors or actuators.

A vibration isolation system is called active if it uses external power to perform its function. It consists a servomechanism with a sensor, actuator, signal processor. Active control systems are required in applications where passive vibration control is not possible because of material constraints or simply not sufficient for the level of control required. Active control is a favorable method of control because it works in a wide frequency range, reducing resonant vibrations within that range and because it is adaptive to changes in the nature of the disturbance. The relationship between these structure types is clearly explained in the following fig.

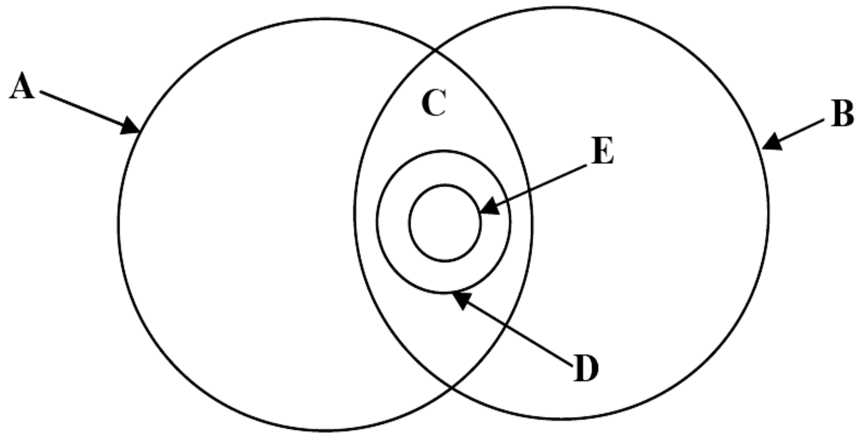


Figure 1. Classification of smart structures

(A) Sensory Structures: These structures possess sensors that enable the determination or monitoring of system states/ characteristics.

(B) Adaptive Structures: These structures possess actuators that enable the alteration of system states or characteristics in a controlled manner.

(C) Controlled Structures: These result from the intersection of the sensory and the adaptive structures. These possess both sensors and actuators integrated in feedback architecture for the purpose of controlling the system states or characteristics.

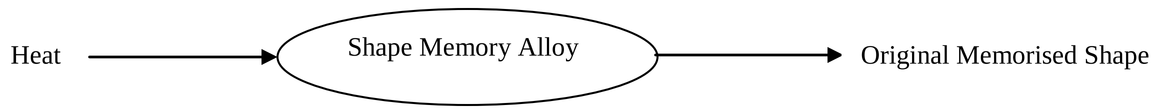
(D) Active Structures: These structures possess both sensors and actuators that are highly integrated into the structure and exhibit structural functionality in addition to control functionality.

(E) Intelligent Structures: These structures are basically active structures possessing highly integrated control logic and electronics that provides the cognitive element of a distributed or hierarchic control architecture

II. ACTIVE SMART MATERIALS

Active smart materials are those materials which possess the capacity to modify their geometric or material properties under the application of electric, thermal or magnetic field, thereby acquiring an inherent capacity to transducer energy. The active smart materials are Piezoelectric material, Shape memory alloys, Electro-rheological fluids and Magneto-structive materials. Being active they can be used as force transducers and actuators. The materials which are not active under the application of electric, thermal or magnetic field are called Passive smart materials. Fiber optic material is good example of passive smart material. Such materials can act as sensors but not as actuators and transducers.

a) SHAPE MEMORY ALLOY'S (SMA)

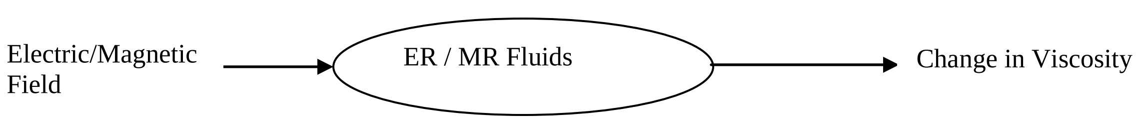


A shape memory alloy (SMA) is able to memories and recover its original shapes after deformed by heating over its transformation temperature. During this transformation large forces or large deformation are generated which can be used for actuation. There are main two types of alloy exhibit a strong shape memory effect copper alloys (Cu-Zn-Al and Cu-Al-Ni) and Nitinols (Ni-Ti) (Nickel – Titanium Alloy)

The main drawbacks of SMA based actuators are comparatively slow response time. This problem is inherent since these alloys rely on heating and cooling for their actuation. Therefore SMA's are not suitable for high frequency control.

SMA made eyeglass frame, Antenna of mobile phone, SMA wire are used as passive energy dissipater to increase the hysteresis damping in structure under earthquake. In advanced application in robotics artificial arms, for buckling control, in medical instruments such as vascular stents and filters.

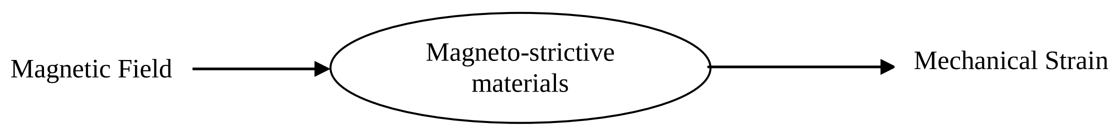
b) Electro/Magneto-Rheological Fluid: (ER / MR Fluids)



ER and MR fluids materials that respond to an applied electric or magnetic field respectively with a important change in Rheological behavior. This is more a semi-smart behavior in the sense that the application of a third party field (Electric or Magnetic) will act on a classical coupling (Viscosity) and that there is no reciprocal effect. These fluids are non-colloidal suspension of polarisable small particles. Their essential characteristic is their ability to reversibly change from a free-flowing, linear viscous liquid to a semi solid with controllable yield strength in milliseconds when exposed to a electric or magnetic field. In absence of an applied field, controllable fluids are reasonably well approximated as Newtonian liquids.

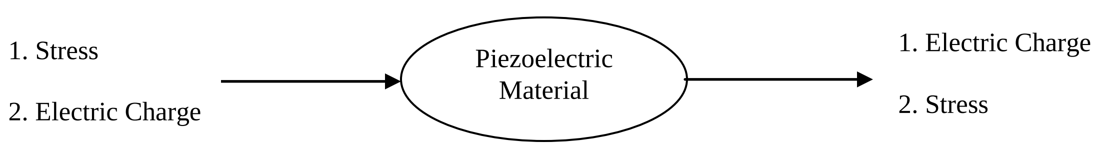
These fluids provides simple, quiet, rapid-response interfaces between electronic controls and mechanical systems. These fluids are used as fast acting, fluid valves with no moving parts in semi-active vibration control system. It is noted that the maximum shear stress obtainable using MR fluids is about 20 times bigger than the maximum shear stress obtainable using ER fluids.

c) Magneto-strictive materials



Magnetostriction is the process by which a ferromagnetic material transforms from one shape to another in the presence of magnetic field. Most ferromagnetic materials exhibit some measurable magnetostriction. Conversely if an external force produce a strain in magneto-strictive material, the materials magnetic state is change. Due to bi-directional effect magneto-strictive materials is used for both actuation and sensing devices. Magneto-strictive materials operates when a compressive load is applied to the material, due to the magneto-elastic coupling forces the domain structure to orient perpendicular to the applied force. Then as a magnetic field is introduced, the domain structure rotates producing the maximum possible strain in the material. Terfenol-D8 (Alloy of the form Tbx Dyl – Xfe2) exhibits the greatest magnetostrictive effect. The magnetostrictive properties of Terfenol D-8 depends on the magnetic and mechanical bias conditions. As compressive load is increased, larger values of field bias as well as larger drive field are required. The coupling factor decreases with increasing magnetic and mechanical bias.

d) Piezoelectric material



Piezoelectricity is the ability of a material to develop an electric charge when subjected to a mechanical strain, this effect is called Direct Piezoelectric Effect (DPE) and Conversely material develop mechanical strain in response to an applied electric field, this effect is called Converse

Piezoelectric Effect (CPE). Due to this coupled mechanical and electrical properties, piezoelectric materials make them well suited for use as sensors and actuators. Sensors use Direct Piezoelectric Effect (DPE) and actuators use Converse Piezoelectric Effect (CPE). As a sensors, deformations cause by the dynamic host structure produce an electric change resulting in an electric current in the sensing circuit. While as an actuators, a high voltage signal is applied to piezoelectric device which deforms the actuator and transmit mechanical energy to the the host structure. Piezoelectric materials basically divided into two group Piezo-ceramics and piezo-polymers.

Piezo-ceramics: The most common commercial piezo-polymer is Barium Titanate ($BaTiO_3$), Lead Titanate ($PbTiO_3$), Lead Zirconate (($PbZrO_3$) Lead metaniobate ($PbNb_2O_6$) and Lead (plumbum) Zirconate Titanate (PZT) [$Pb(ZrTi)O_3$]. Among these materials last Lead (plumbum) Zirconate Titanate (PZT) become the dominant piezo-electric ceramic material for transducer due to its high coupling coefficient (0.65). When this PZT plate subjected to static or dynamic loads, it can generate voltages as high as 20,000 volts.

Examples: Microphones, headphones, loudspeakers, buzzers, wrist watches, clocks, calculators, hydrophones and projectors.

Piezo-polymers: The most common commercial piezo-polymer is polyvinvylidene Fluoride (PVDF). It is made up of long chains of the repeating monomer (- Ch_2 – Cf_2 -) each of which has an inherent dipole moment. Both PZT and PVDF are usually produced in the thin sheets with film of metal deposited on the opposite surface to form electrodes. Piezo-polymers are tough and flexible they have small stiffness due to this PVDF are good candidates for sensing of their small stiffness while Piezo-ceramics are brittle and stiff and having greater elastic modulus for effective mechanical coupling to the structure hence it is better suited for actuators. Hence PVDF used as Sensors and PZT used as Actuators.

Pizoelectric materials are works in both way (DPE and CPE) and also applicable for low as well as high frequency response. These materials are following Features/Advantages/Properties of PZT materials over remaining three materials.

1) The materials has resilience property, light in weight and high bandwidth of devices.
2) Unlimited resolution: PZT actuators makes motion in the sub-nanometer range. Hence they have no moving parts in contact with each other to limit resolution.

3) No wear and tear: PZT materials has no moving parts like gears, bearings hence no wear and tear are shows.
4) Fast expansion: This material react in a matter of microseconds. Acceleration rates of this material are more than 10,000 x g.
5) High force generation: The PZT materials are capable for the moving loads of several tones. In this case large masses can be moved and positioned accurately.
6) No magnetic field: PZT material do not produce magnetic field nor affected by the magnetic field hence suited for application where magnetic field cannot be tolerated.
7) Low power consumption: This materials are holding heavy loads for long periods consumes virtually no power/very less power.
8) Operation at cryogenic temperature: The PZT materials are effected continuously even at temperature close to 0 Kelvin.

By applying properties of PZT materials some researchers utilized the PZT materials in precision control of dynamical systems due to their special characteristics. Some are used the PZT materials as elements of an intelligent structure. They introduce an analytical model and compared it with an experimental set. They performed a scaling analysis to demonstrate that the effectiveness of PZT actuators are independent of the size of the structure and to evaluate various PZT materials base on their effectiveness in transmitting strain to the substructure. Many researchers are used a PZT actuator in a fixed position, the first two modes of vibration for a beam were controlled. They used single actuator to suppress vibrations by using acceleration feedback controllers. Knowing the frequencies for a smart structure helps in designing the parameters of controller. Some are explained that the PZT ceramics are used for reconstruction filters.

III. VIBRATION ANALYSIS OF CANTILEVER BEAM

In this paper, a simple smart system with a PZT sensor and a PZT actuator is tested for vibration Analysis. The length, width and depth of the beam are taken as 0.08, 0.001 and 0.016 m, respectively. For the PZT sensor and actuator, the length, width and depth are taken as 0.01, 0.0004 and 0.016 m, respectively. The material properties are listed in Table 1.

Table 1:- Material Properties for Piezoceramic and Steel plate

Piezoceramic (BM500)	**Values**
Elasticity matrix at constant electric field, c_{11} (Pa)	1.26×10^{11}
Elasticity matrix at constant electric field, c_{12} (Pa)	8.41×10^{11}
Elasticity matrix at constant electric field, c_{22} (Pa)	1.17×10^{11}
Elasticity matrix at constant electric field, c_{33} (Pa)	1.26×10^{11}
Piezoelectric constant matrix, e_{21} (C/m^2)	-5.4
Piezoelectric constant matrix, e_{22} (C/m^2)	15.8
Dielectric matrix at constant strain, ε_{11}(F/m)	1.151×10^{-3}
Dielectric matrix at constant strain, ε_{22} (F/m)	1.043×10^{-3}
Mass density, ρ (kg/m^2)	7800
Beam (Steel)	
Young modulus, Y (Pa)	2.07×10^{11}
Poission's ratio, γ	0.3
Mass density , ρ (kg/m^3)	7800

The model of cantilever beam will be done using commercial FE software (ANSYS). In ANSYS, the beam is modeled with a 2-D elastic beam element (BEAM3), and the PZT actuator and sensor are modeled using a 2-D coupled field element (PLANE13).

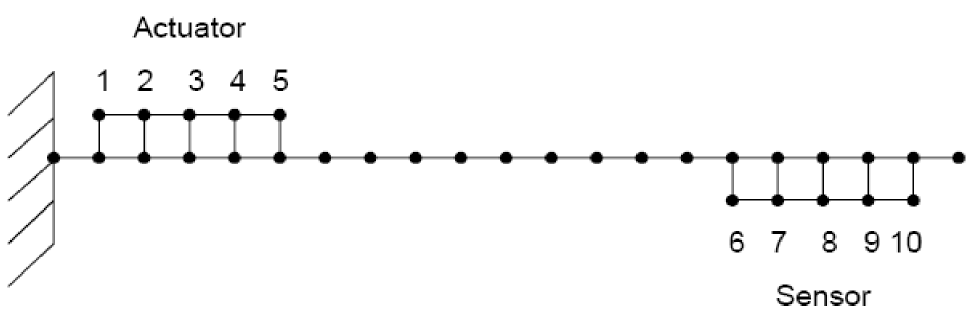


Figure 2. Finite Element Mesh of the Beam and PZT's System

Before we start the analysis in ANSYS, we have to check that our actuator locations do not interface with the modal nodes. That is, the actuator node where we feed the input voltages should not be on the same location of the modal nodes, i.e. transverse displacement should not be zero. Figures 3 to 5 show the first three mode shapes for the beam with and without the PZT actuator.

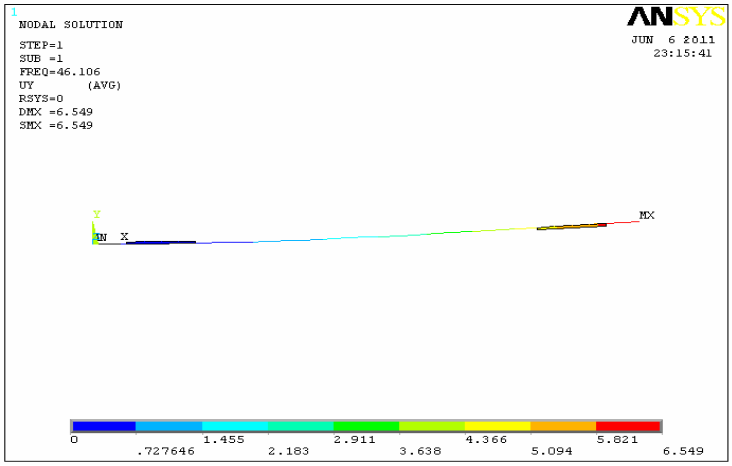


Figure 3. 1st mode shape for the beam with Piezoelectric material.

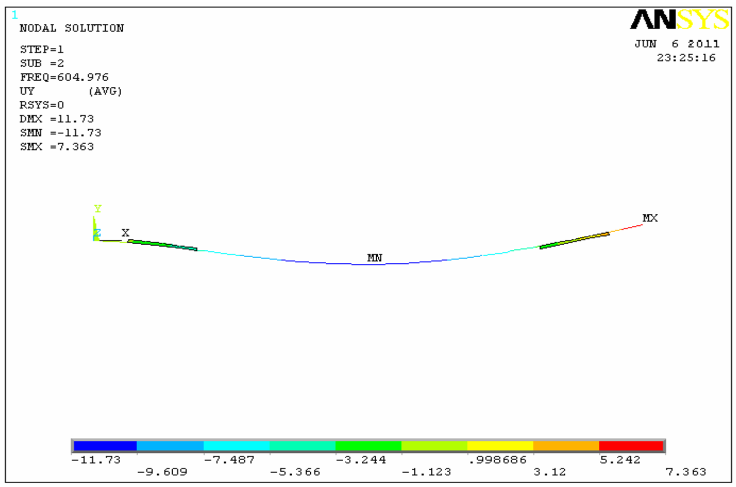


Figure 4. 2nd mode shape for the beam with Piezoelectric material.

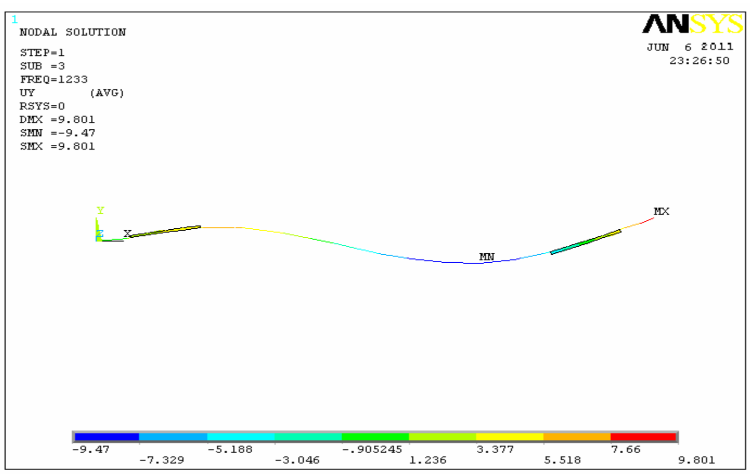


Figure 5. 3rd mode shape for the beam with Piezoelectric material.

Table-2 shows the values for the natural frequencies and transverse displacement at node 5 for the first three mode shapes at each PZT actuator location.

TABLE 2:-Transverse displacements for node 5 of the actuator for the first three mode shapes of the beam with PZT's.

Location (mm)	Mode-shape	Frequency (Hz)	Displacement (mm)
5	1st mode	40.016	0.32141
5	2nd mode	604.97	-4.9201
5	3rd mode	1233	5.5120
15	1st mode	44.336	0.95776
15	2nd mode	385.52	-6.0748
15	3rd mode	1197	1.3826
20	1st mode	132.41	3.7923
20	2nd mode	411.98	-4.3288
20	3rd mode	1394	-3.1248
25	1st mode	42.301	1.7350
25	2nd mode	304.15	-5.5271
25	3rd mode	1362	-1.4421
35	1st mode	39.748	2.5323
35	2nd mode	292.52	-4.6329
35	3rd mode	1545	-3.8977
40	1st mode	81.230	5.5361
40	2nd mode	534.62	0.02037
40	3rd mode	1165.8	-2.4540
45	1st mode	36.742	3.2465
45	2nd mode	338.40	-3.2314
45	3rd mode	1663	-4.6054
60	1st mode	51.094	5.9164
60	2nd mode	638.85	3.7533
60	3rd mode	1270.7	4.3589

Table-3 shows the values for the first three natural frequencies obtained from ANSYS and MATLAB for the cantilever beam with piezoelectric actuators and sensors. From this point we can start the study with the model we did in ANSYS.

TABLE 3:-First three natural frequencies

Mode shape	Natural frequency with Pizoelectric Material	
	MATLAB	ANSYS
1st mode	63.45	63.53
2nd mode	410.45	410.96
3rd mode	1153.11	1153.3

From table 2, in all the cases, node 5 has values for the transverse displacement other than zero. So we can conclude from the table that the actuator can be placed in all those locations.

IV. VIBRATION ANALYSIS OF CANTILEVER PLATE

In this case, a smart system consisting of a plate fixed at one side and mounted with a PZT sensor and a PZT actuator is tested for vibration control. The length, width and thickness of the plate are 0.1, 0.03 and 0.002 m, respectively. For the PZT sensor and actuator, the length, width and thickness are taken as 0.02, 0.01 and 0.002 m, respectively. Figure 6 shows the plate with PZT actuator and sensor. The material properties are listed in Table 4.1.

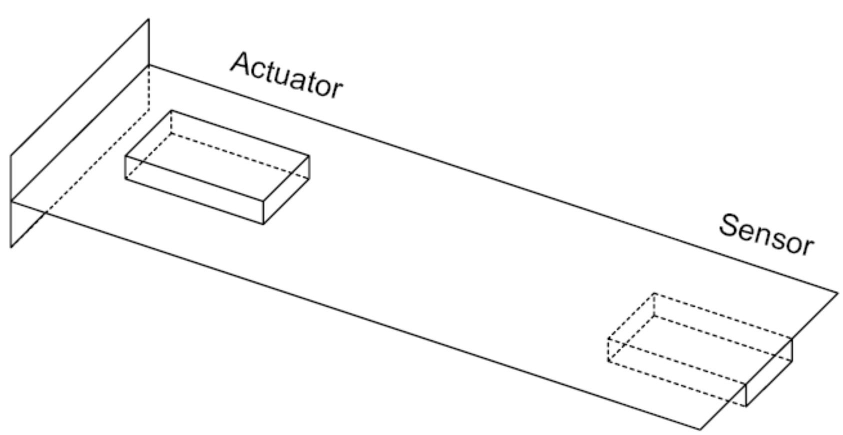


Figure 6. Smart structure consisting of steel plate, PZT sensor and PZT actuator

Table 4: Material Properties for the plate with PZT's

Piezoceramic (BM500)	**Standard Values**
C_{11} (Pa)	1.26×10^{11}
C_{12} (Pa)	8.41×10^{10}
C_{13} (Pa)	7.95×10^{10}
C_{22} (Pa)	1.17×10^{11}
C_{23} (Pa)	8.41×10^{10}
C_{33} (Pa)	1.26×10^{11}
C_{44} (Pa)	2.30×10^{10}
C_{55} (Pa)	2.30×10^{10}
C_{66} (Pa)	2.35×10^{10}
e_{12} (C/m^2)	-5.4
e_{22} (C/m^2)	15.8
e_{32} (C/m^2)	-5.4
e_{41} (C/m^2)	12.3
e_{53} (C/m^2)	12.3
ε_{11} (F/m)	1.151×10^{-3}
ε_{22} (F/m)	1.043×10^{-3}
ε_{33} (F/m)	1.151×10^{-3}
ρ (Kg/m^3)	7800
Beam(Steel)	
Y (Pa)	2.07×10^{11}
υ	0.3
ρ (Kg/m^3)	7800

Since we have five mechanical DOF for each node of the plate, the total mechanical DOF of the plate will be 236 (the nodes at the fixed side have zero DOF). Having the PZT's grounded at the boundary with the plate gives a total of 12 electrical DOF. In ANSYS, the plate is modeled using an elastic 4-noded element (SHELL63) and the PZT's are modeled using a 3-D coupled element (SOLID5). The actuator are placed in six different positions as shown in Figure 7. Note that each square is 0.01 x 0.01 m in size.

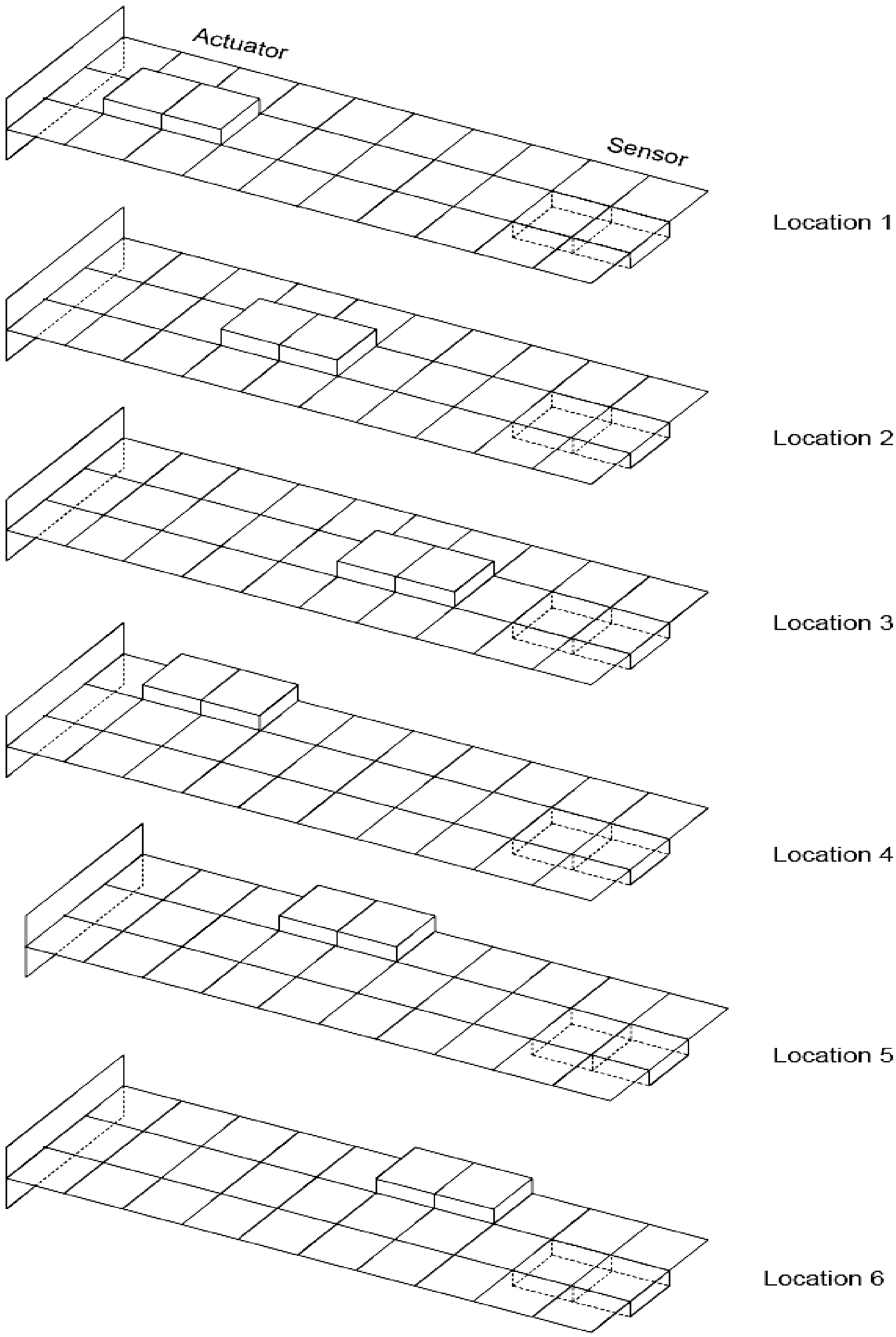


Figure 7. Different location for Piezoelectric actuator placements

Table 5: Transverse displacements for nodes 3 and 6 of the actuator for the first seven mode shapes of the beam with PZT's. (Location 1)

Mode Shape	**Natural Frequency (Hz)**	**Node**	**Displacement**
1st mode	159.05	3	1.1435
1st mode	159.05	6	1.1435
2nd mode	999.072	3	-4.6965
2nd mode	999.072	6	-4.6965
3rd mode	1181.00	3	-1.3187
3rd mode	1181.00	6	-1.3187
4th mode	2135.00	3	0.0039
4th mode	2135.00	6	-0.0039
5th mode	2808.00	3	6.4518
5th mode	2808.00	6	6.4518

Table 6: Transverse displacements for nodes 3 and 6 of the actuator for the first seven mode shapes of the beam with PZT's. (Location 2)

Mode Shape	**Natural Frequency (Hz)**	**Node**	**Displacement**
1st mode	154.22	3	2.8706
1st mode	154.22	6	2.8706
2nd mode	983.45	3	6.1449
2nd mode	983.45	6	-6.1449
3rd mode	1175.6	3	-2.4985
3rd mode	1175.6	6	2.4985
4th mode	2118.6	3	0.0756
4th mode	2118.6	6	0.0756
5th mode	2848.9	3	0.0693
5th mode	2848.9	6	0.0693

Table 7: Transverse displacements for nodes 3 and 6 of the actuator for the first seven mode shapes of the beam with PZT's. (Location 3)

Mode Shape	**Natural Frequency (Hz)**	**Node**	**Displacement**
1st mode	149.37	3	4.8474
1st mode	149.37	6	4.8774
2nd mode	1006.5	3	2.9008
2nd mode	1006.5	6	2.9008
3rd mode	1153.3	3	-3.3363
3rd mode	1153.3	6	3.3363
4th mode	2079.6	3	0.0437
4th mode	2079.6	6	-0.0437
5th mode	2881.3	3	5.5822
5th mode	2881.3	6	5.5822

Table 8: Transverse displacements for nodes 3 and 6 of the actuator for the first seven mode shapes of the beam with PZT's. (Location 4)

Mode Shape	**Natural Frequency (Hz)**	**Node**	**Displacement**
1st mode	158.55	3	1.1457
1st mode	158.55	6	1.1105
2nd mode	993.06	3	-4.7379
2nd mode	993.06	6	-4.9304
3rd mode	1172.8	3	1.2876
3rd mode	1172.8	6	3.8390
4th mode	2162.3	3	0.2828
4th mode	2162.3	6	0.3559
5th mode	2795.9	3	6.65
5th mode	2795.9	6	7.6441

Table 9: Transverse displacements for nodes 3 and 6 of the actuator for the first seven mode shapes of the beam with PZT's. (Location 5)

Mode Shape	**Natural Frequency (Hz)**	**Node**	**Displacement**
1st mode	153.88	3	2.8688
1st mode	153.88	6	2.8449
2nd mode	975.00	3	-6.3801
2nd mode	975.00	6	-7.4088
3rd mode	1150.4	3	1.0753
3rd mode	1150.4	6	5.7993
4th mode	2127.4	3	-0.1163
4th mode	2127.4	6	-0.2133
5th mode	2838.4	3	0.1304
5th mode	2838.4	6	0.5438

Table 10: Transverse displacements for nodes 3 and 6 of the actuator for the first seven mode shapes of the beam with PZT's. (Location 6)

Mode Shape	**Natural Frequency (Hz)**	**Node**	**Displacement**
1st mode	149.29	3	4.8801
1st mode	149.29	6	4.8737
2nd mode	987.46	3	-3.5840
2nd mode	987.46	6	-5.8057
3rd mode	1112.6	3	1.6176
3rd mode	1112.6	6	7.5023
4th mode	2078.6	3	0.0181
4th mode	2078.6	6	-0.0562
5th mode	2876.9	3	-5.3690
5th mode	2876.9	6	-5.6585

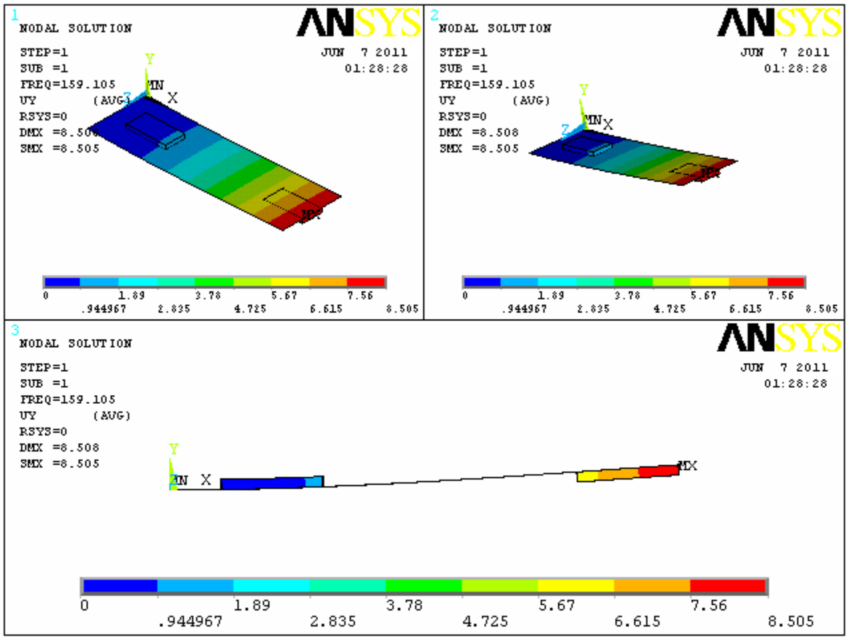


Figure 8. 1st mode shape for the plate with PZT's
(PZT actuator placed at location 1).

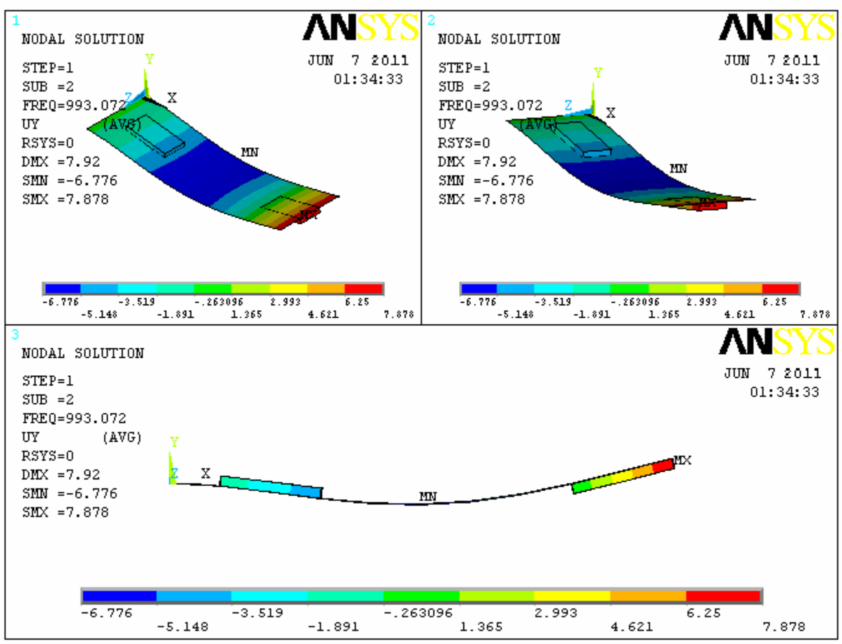


Figure 9. 2nd mode shape for the plate with PZT's
(PZT actuator placed at location 1).

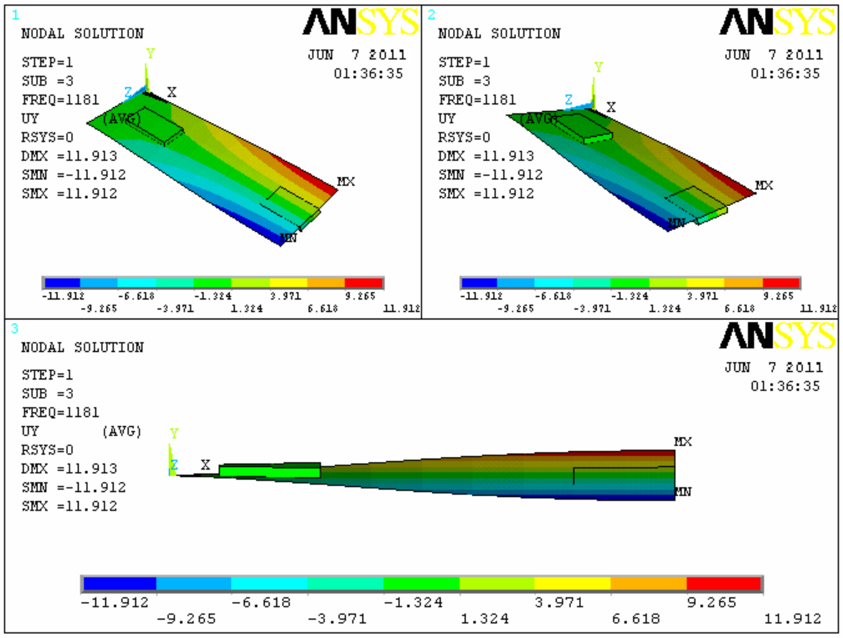


Figure 10. 3rd mode shape for the plate with PZT's
(PZT actuator placed at location 1).

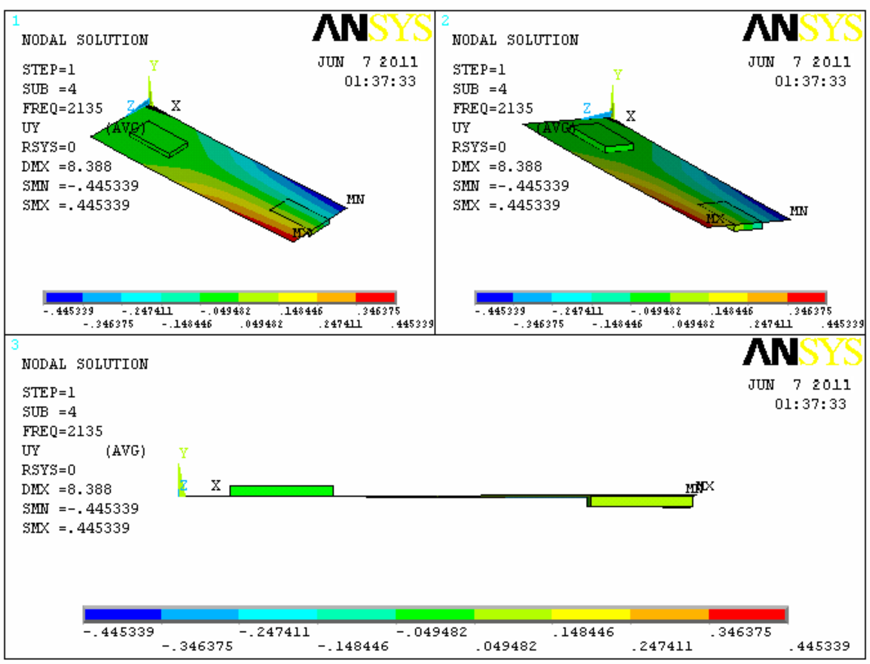


Figure 11. 4th mode shape for the plate with PZT's
(PZT actuator placed at location 1).

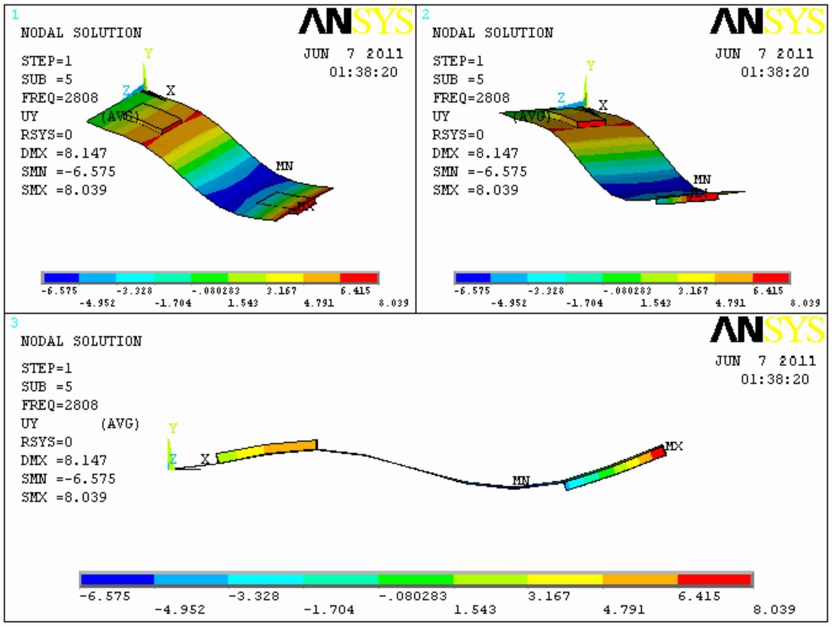


Figure 12. 5th mode shape for the plate with PZT's (PZT actuator placed at location 1).

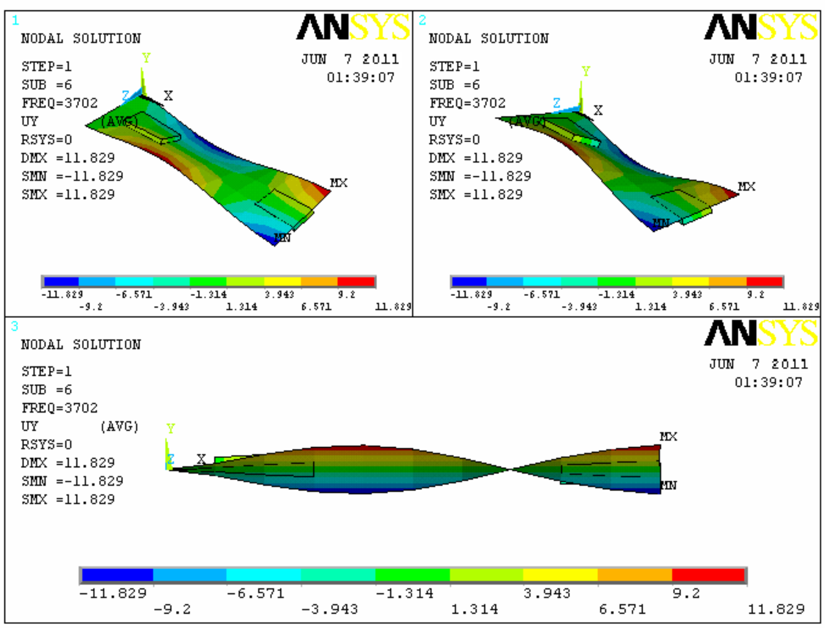


Figure 13. 6th mode shape for the plate with PZT's (PZT actuator placed at location 1).

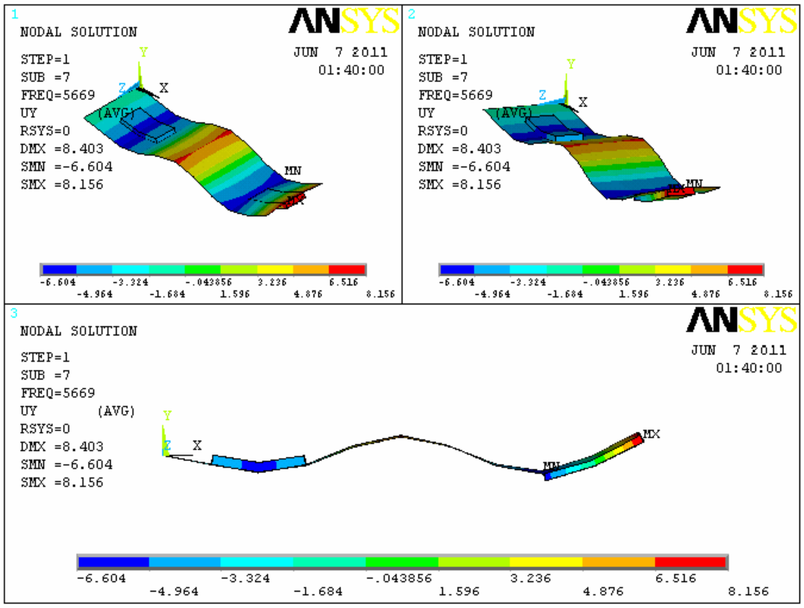


Figure 14. 7th mode shape for the plate with PZT's
(PZT actuator placed at location 1).

Tables 5 – 10 shows the values for the transverse displacements for the actuator nodes 3 and 6 at each location for the first seven natural frequencies. Figures 8 – 14 show the first seven mode shapes for the plate with the PZT's for the first actuator location, that is location 1.

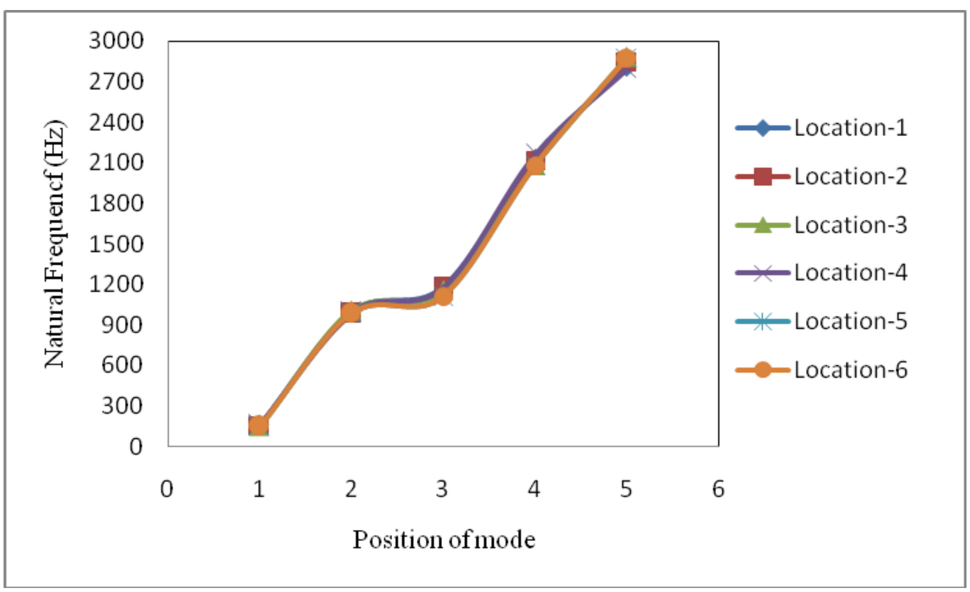


Figure 15. Natural frequencies for different locations of Piezoelectric Actuator.

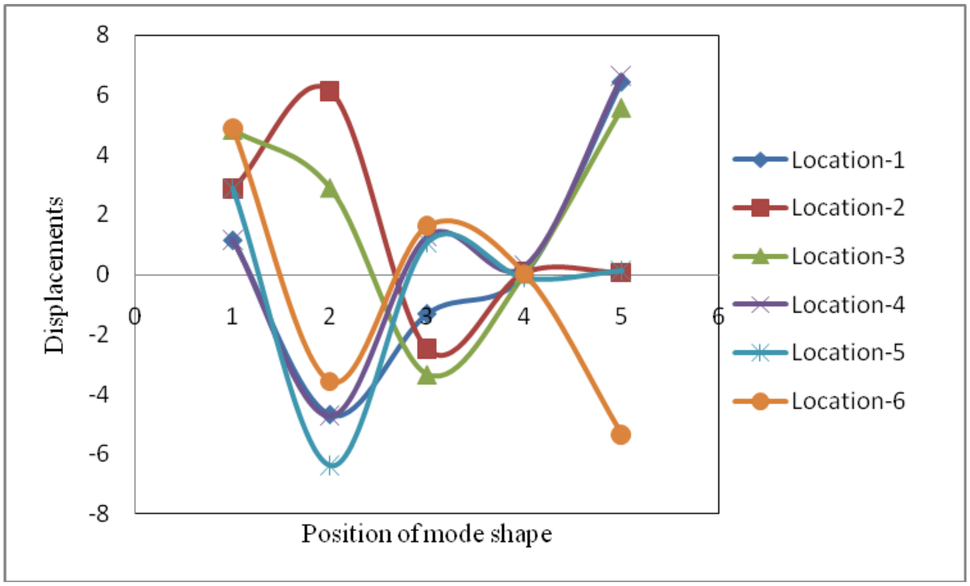


Figure 16. Displacements for different location of Piezoelectric Actuator

Figure 15 and 16 shows that the natural frequencies and displacements for different locations of piezoelectric actuator. The natural frequencies are slightly change for different locations but the displacements changes are in large magnitudes. This shows that if the position of piezoelectric actuator are change then the vibration parameters are also changes.

V. CONCLUSIONS

In this a comprehensive study of smart materials and smart structures is done. For the effect of the piezoelectric actuator placement on controlling the structural vibrations. Two systems were used for this study, the first one was a 2-D beam with PZT actuator and sensor, and the second one was a 3-D plate with PZT actuator and sensor. Both systems were modeled in ANSYS. All the cases, for both systems, showed that our actuator locations used for the study were acceptable since the feeding node had transverse displacement values that were not zero. For the 2-D beam, the modal nodes were checked up to the 3rd mode shape, whereas for the 3-D plate we checked up to the 5th mode shape.

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